

Aspects of topologically gauged M2-branes with six supersymmetries: towards a "sequential AdS/CFT"?

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ABSTRACT: In this note we review the construction of topologically gauged M2 branes with 6 supersymmetries and discuss some of its properties. This is done using the 3-algebra formulation thereby covering all possible gauge groups. We will elaborate upon 1) the fundamental identity and its solutions noting, provided these gauged theories describe stacks of branes, the case of a single brane, 2) the chiral point solution to the field equations (occurring even for a single brane) that breaks the superconformal symmetries down to those of AdS_3 (TMG) supergravity, 3) physical parameters and how they scale in the compactification from 11d to 10d and give rise to matter theories in curved space-times, and finally 4) a more speculative comment on "*sequential AdS/CFT*". Here we propose that the superconformal symmetry breaking in topologically gauged theories leads to the sequence $AdS_4/CFT_3 \rightarrow AdS_3/CFT_2$ and that the higgsing in the 3d boundary theory is related to a change of foliation in the AdS_4 bulk theory.

KEYWORDS: String theory, M-theory, Branes, Chern-Simons theory, AdS/CFT.

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1. Introduction and summary

At the IR fix-point stacks of M2-branes are described by interacting superconformal Chern-Simons(CS) matter theories [1]. Such theories were first discovered with eight supersymmetries by Bagger and Lambert and independently by Gustavsson (BLG) [2, 3, 4, 5] and were shown to have a number of interesting properties. It was, however, soon realized that this (classical) theory has the problem that it is heavily constrained with only one possible solution of the fundamental identity, or in other words the theory exists only for one gauge group, namely $SO(4)$. Although a level k can be introduced (by rescaling the four-index structure constants) and argued to take any integer value, only $k = 1, 2$ have a possible interpretation in terms of stacks of M2-branes. One is therefore forced in the M-theory context to deal with a strongly coupled system since the weak coupling limit requires taking k large. The BLG theory seems for this reason to describe only stacks of two branes although more recent work involving monopole operators [6] has produced a number of new quantum theories with eight supersymmetries, see e.g. [7] and references therein.

Reducing the number of supersymmetries to six gives rise to the so called ABJM theories [6] which already at the classical level are much more general and can describe stacks with any number of branes and with a level that can take any integer value. In this case the interpretation in terms of stacks of M2-branes is clear and so is its connection to eleven dimensional supergravity and M-theory compactified on $AdS_4 \times S^7/\mathbf{Z}_k$ where k is the level in the ABJM *CFT* [6] which is based on the construction in [8]. The much looser structure found here makes the generalization to other gauge groups than those discussed by ABJM possible [9]. The admissible gauge groups have been completely classified using a variety of methods, see [10, 11, 12, 13]. In particular the work by Palmkvist [13] is relevant

here since the method used there is based on the three-algebra formulation of the $N = 6$ theories.

The above superconformal M2-brane theories have apart from the gauge symmetries associated with the Chern-Simons (CS) vector gauge fields also global symmetries corresponding to the superconformal group namely translations, supersymmetry and their *special* conformal counterparts plus Lorentz rotations, dilatation and R-symmetries. We will here describe how these global symmetries can be made local by introducing conformal supergravity into the game and couple it to the BLG [14] and ABJM [16] CS matter theories. While the construction in the case of BLG met with difficulties and could not be carried through completely in [14]¹, the complete lagrangian for the ABJM case has, however, been derived in [16]. As we will see below it is found to have new potential terms for the scalar fields over and above those present in the ungauged ABJM theory. This leads to some very interesting new properties in particular one finds a Higgs effect with an intriguing end result related to chiral gravity [16].

After a brief discussion of the topologically gauged theories and how they are constructed, we turn to some of their special properties related to the new potential terms that were found in the ABJM topologically gauged theory in [16] and worked out in more detail in some special cases in [17]. We will see that the new structure of the scalar potential leads to a Higgs effect that breaks the superconformal symmetries to those of AdS_3 corresponding to a compactification from M-theory in eleven dimensions to string theory in ten. The end-result is then a non-trivial interacting CS matter theory in a curved background. We will also try to argue that theories of this type may arise from AdS_4/CFT_3 based on Neumann boundary conditions as has been advocated, e.g., in work by Marolf, de Haro and others [18, 19, 20, 21]. Since the presence of new terms in the scalar field potential leads to a Higgs effect which breaks the conformal symmetries to those of AdS_3 it suggests the possibility that a second AdS/CFT comes into play relating the theory in AdS_3 to a CFT_2 , something that might be called "sequential AdS/CFT ":

$$AdS_4(N)/CFT_3(TG) \rightarrow AdS_3(H)/CFT_2 \quad (1.1)$$

where N refers to Neumann boundary conditions, TG to topologically gauged BLG or ABJM theory and H to its higgsed version. The final CFT_2 is unknown. As we will see in the discussion of the structure constants later this scenario is also possible for the case of a single brane. The assumptions needed for this sequential AdS/CFT to work will be discussed in more detail in a later section.

A similar idea was put forward some time ago by M. Vasiliev [22]² based on properties of higher spin (HS) algebras but without having any theory that explicitly can generate the necessary breaking from conformal to AdS symmetries as we have in the present work. It might be argued that going to the boundary of AdS twice is impossible because the boundary of a boundary is normally zero. A possible way out of this problem might be to relate the higgsing described above to a change of foliation of the AdS_4 space. Indeed

¹The complete theory has now been constructed [15].

²I am grateful to M. Vasiliev for discussions about these ideas.

it is well-known that AdS spaces can be foliated in many ways some of them leading to leaves that are themselves AdS spaces, see e.g. [23, 24]. A recent discussion of phenomena of this kind can be found in [25] where also the possibility of a sequential AdS/CFT is mentioned. This is based on previous work in [19].

2. Construction of topologically gauged M2 brane theories with six supersymmetries

The title of this section refers to the theories obtained by gauging all global symmetries of the ABJM/ABJ type theories [6, 9] keeping the superconformal symmetries intact but turned into local ones. This can be achieved by coupling, for instance using a Noether procedure, the Chern-Simons-like theory for $N = 6$ conformal supergravity to the ABJM/ABJ CS matter theories as first done in the BLG case in [14]. The only non-trivial aspect of this construction is that, in order to get a supersymmetric lagrangian, one has to add a $U(1)$ gauge field not a priori present in either the on-shell superconformal supergravity lagrangian or in the ABJM/ABJ matter theories. The details of the whole construction can be found in [16]³. It should be noted that the topological gauging leads to the an interacting theory even if we start from a free theory of scalars and spinors. In particular the CFT for a single brane will start to self-interact when it is gauged.

The on-shell supergravity fields are the three gauge fields of 'spin' 2, 3/2 and 1, i.e.

$$e_\mu{}^\alpha, \chi_{\mu AB}, B_\mu^A{}_B \quad (2.1)$$

The capital Latin indices used here are in the fundamental of the R-symmetry group $SU(4) \times U(1)$ and the corresponding gauge field $B_\mu^A{}_B$ is in the adjoint of $SU(4)$ while the $U(1)$ factor plays no role in the gravity sector. It might be argued that the extra gauge field C_μ that one is forced to introduce in the coupling to matter is just the gauge field of this $U(1)$ factor. It is in fact part of the off-shell multiplet [26], see also [27]. The spin 3/2 field $\chi_{\mu AB}$, and the supersymmetry parameter ϵ_{AB} , are both antisymmetric and self-dual in the two indices AB in order to accommodate exactly six supersymmetries. The matter sector contains the fields

$$Z_a^A, \Psi_{Aa}, \tilde{A}_\mu{}^a{}_b = A_\mu{}^d{}_c f^{ac}{}_{bd} \quad (2.2)$$

where one should remember that the complex conjugation acts on all the indices⁴ as $(Z_a^A)^* = \bar{Z}_A^a$ and $(\Psi_{Aa})^* = \Psi^{Aa}$, see [28]. The three-algebra formulation was originally obtained from the ABJM quiver version in [29]. While the BLG and ABJM Chern-Simons theories are well studied in a large number of papers that have appeared since their introduction in 2007, the conformal supergravity theories that we need here are perhaps

³The topologically gauged lagrangian below has so far been derived only in this way. However, there are still some mult fermionic terms in the variation of the lagrangian that have not been verified to cancel. Although it is very unlikely that something could go wrong at the final steps of the construction it would be welcome to have an alternative derivation in order to prove that the theory exists. Work in this direction is currently under way [15]

⁴The space-time spinors are not affected by complex conjugations since they are Majorana.

less familiar. These were originally constructed in the 1980's by Deser and Kay [30], van Nieuwenhuizen [31] for $N = 1$, and by Lindström and Roček [32] for arbitrary N .

Using the Noether procedure we find that the coupled lagrangian of the topologically gauged theory is given by (with $A^2 = \frac{1}{2}$) [16, 17]

$$\begin{aligned}
L = & g_M^{-2} L_{sugra}^{conf} + L_{ABJM}^{cov} + \frac{1}{2g_M^2} \epsilon^{\mu\nu\rho} C_\mu \partial_\nu C_\rho \\
& + iAe\bar{\chi}_\mu^{BA} \gamma^\nu \gamma^\mu \Psi_{Aa} (\tilde{D}_\nu \bar{Z}_B^a - \frac{i}{2} A \bar{\chi}_{\nu BC} \Psi^{Ca}) + c.c. \\
& + i\epsilon^{\mu\nu\rho} (\bar{\chi}_\mu^{AC} \chi_{\nu BC}) Z_a^B \tilde{D}_\rho \bar{Z}_A^a + c.c. \\
& - iA(\bar{f}^{\mu AB} \gamma_\mu \Psi_{Aa} \bar{Z}_B^a + \bar{f}_{AB}^\mu \gamma_\mu \Psi^{Aa} Z_a^B) \\
& - \frac{e}{8} \tilde{R} |Z|^2 + \frac{i}{2} |Z|^2 \bar{f}_{AB}^\mu \chi_\mu^{AB} \\
& + 2i\lambda A e f_{cd}^{ab} (\bar{\chi}_{\mu AB} \gamma^\mu \Psi^{d[B]} Z_a^D Z_b^A \bar{Z}_D^c + c.c.) \\
& - i\lambda \epsilon^{\mu\nu\rho} (\bar{\chi}_{\mu AB} \gamma_\nu \chi_\rho^{CD}) (Z_a^A Z_b^B \bar{Z}_C^c \bar{Z}_D^d) f_{cd}^{ab} \\
& + \frac{i\lambda}{4} \epsilon^{\mu\nu\rho} (\bar{\chi}_{\mu AB} \gamma_\nu \chi_\rho^{AB}) (Z_a^C Z_b^D \bar{Z}_C^c \bar{Z}_D^d) f_{cd}^{ab} \\
& - \frac{ig_M^2}{16} e \epsilon^{ABCD} (\bar{\Psi}_{Aa} \Psi_{Bb}) \bar{Z}_C^a \bar{Z}_D^b + c.c. \\
& + \frac{ig_M^2}{16} e (\bar{\Psi}_{Db} \Psi^{Db}) |Z|^2 - \frac{ig_M^2}{4} e (\bar{\Psi}_{Db} \Psi^{Bb}) \bar{Z}_B^a Z_a^D \\
& + \frac{ig_M^2}{8} e (\bar{\Psi}_{Db} \Psi^{Da}) \bar{Z}_B^b Z_a^B + \frac{3ig_M^2}{8} e (\bar{\Psi}_{Db} \Psi^{Ba}) \bar{Z}_B^b Z_a^D \\
& - \frac{ig_M^2}{16} e A (\bar{\chi}_{\mu AB} \gamma^\mu \Psi^{Bb}) |Z|^2 Z_b^A - \frac{ig_M^2}{4} e A (\bar{\chi}_{\mu AB} \gamma^\mu \Psi^{Db}) Z_a^A Z_b^B \bar{Z}_D^a + c.c. \\
& - \frac{ig_M^2}{4} \epsilon^{\mu\nu\rho} (\bar{\chi}_{\nu AB} \gamma_\rho \chi_\mu^{CD}) Z_a^A Z_b^B \bar{Z}_C^a \bar{Z}_D^b + \frac{ig_M^2}{64} \epsilon^{\mu\nu\rho} (\bar{\chi}_{\nu AB} \gamma_\rho \chi_\mu^{AB}) |Z|^4 \\
& + \frac{\lambda g_M^2}{8} e f_{cd}^{ab} |Z|^2 Z_a^C Z_b^D \bar{Z}_C^c \bar{Z}_D^d + \frac{\lambda g_M^2}{2} e f_{cd}^{ab} Z_a^B Z_b^C Z_e^D \bar{Z}_B^e \bar{Z}_C^c \bar{Z}_D^d \\
& + \frac{5g_M^4}{12 \cdot 64} e (|Z|^2)^3 - \frac{g_M^4}{32} e |Z|^2 Z_b^A Z_a^C \bar{Z}_C^b \bar{Z}_A^a + \frac{g_M^4}{48} e Z_a^A Z_b^B Z_d^C \bar{Z}_A^B \bar{Z}_B^d \bar{Z}_C^a,
\end{aligned}$$

where $|Z|^2$ stands for

$$Z_a^A \bar{Z}_A^a = Tr(\bar{Z}_A, Z^A), \quad (2.3)$$

and *c.c.* refers to complex conjugation of the term on the line where it occurs.

For the two subsectors that are coupled we use the following lagrangians

$$\begin{aligned}
L_{ABJM}^{cov} = & -e(\tilde{D}_\mu Z^A{}_a)(\tilde{D}^\mu \bar{Z}_A{}^a) - \frac{1}{2}(ie\bar{\Psi}^{Aa} \gamma^\mu \tilde{D}_\mu \Psi_{Aa} + ie\bar{\Psi}_{Aa} \gamma^\mu \tilde{D}_\mu \Psi^{Aa}) \\
& - ie\lambda f_{cd}^{ab} \bar{\Psi}^{Ad} \Psi_{Aa} Z_b^B \bar{Z}_B^c + 2ie\lambda f_{cd}^{ab} \bar{\Psi}^{Ad} \Psi_{Ba} Z_b^B \bar{Z}_A^c \\
& - \frac{i}{2}\lambda e \epsilon_{ABCD} f_{cd}^{ab} \bar{\Psi}^{Ac} \Psi^{Bd} Z_a^C Z_b^D - \frac{i}{2}\lambda e \epsilon^{ABCD} f_{cd}^{ab} \bar{\Psi}_{Ac} \Psi_{Bd} \bar{Z}_C^a \bar{Z}_D^b \\
& - eV + \frac{1}{2}\lambda \epsilon^{\mu\nu\lambda} (f_{cd}^{ab} A_\mu{}^d \partial_\nu A_\lambda{}^a + \frac{2}{3}\lambda f^{bd}{}_{gc} f^{gf}{}_{ae} A_\mu{}^a {}_b A_\nu{}^c {}_d A_\lambda{}^e), \quad (2.4)
\end{aligned}$$

$$V = \frac{2}{3} \Upsilon^{CD}{}_{Bd} \bar{\Upsilon}^{CD}{}^{Bd}, \quad (2.5)$$

$$\Upsilon^{CD}{}_{Bd} = \lambda f_{cd}^{ab} Z_a^C Z_b^D \bar{Z}_B^c + \lambda f_{cd}^{ab} \delta^{[C}{}_{B} Z^{D]}{}_a \bar{Z}_E^c \quad (2.6)$$

and

$$\begin{aligned}
L_{sugra}^{conf} = & \frac{1}{2} \varepsilon^{\mu\nu\rho} Tr_\alpha (\tilde{\omega}_\mu \partial_\nu \tilde{\omega}_\rho + \frac{2}{3} \tilde{\omega}_\mu \tilde{\omega}_\nu \tilde{\omega}_\rho) - 2\varepsilon^{\mu\nu\rho} Tr_A (B_\mu \partial_\nu B_\rho + \frac{2}{3} B_\mu B_\nu B_\rho) \\
& - ie^{-1} \varepsilon^{\alpha\mu\nu} \epsilon^{\beta\rho\sigma} (\tilde{D}_\mu \bar{\chi}_\nu^{AB} \gamma_\beta \gamma_\alpha \tilde{D}_\rho \chi_{AB}).
\end{aligned} \quad (2.7)$$

and the action of the covariant derivative on a spinor is given by

$$\tilde{D}_\mu \Psi^{Aa} = \partial_\mu \Psi^{Aa} + \frac{1}{4} \tilde{\omega}_{\mu\alpha\beta} \gamma^{\alpha\beta} \Psi^{Aa} + B_{\mu B}^A \Psi^{Ba} + \lambda \tilde{A}_{\mu b}^a \Psi^{Ab} + q C_\mu \Psi^{Aa}. \quad (2.8)$$

From the work of [16]⁵ we know that the $U(1)$ charge $q = \frac{1}{16}$ in order for the theory to possess six local special conformal as well as ordinary supersymmetries. Furthermore, the level k or its inverse $\lambda = \frac{2\pi}{k}$ can be introduced as usual by rescaling the structure constant but besides that one can also, as done in [17], introduce a dimensionless gravitational coupling constant g_M by rescaling the trace in the three-algebra. Another, perhaps more physical way, to get g_M in the right places in the lagrangian is to, in the Noether construction of [16], start from a pure supergravity lagrangian multiplied by a factor g_M^2 . This will automatically give the correct result.

The supersymmetry variations under which the above lagrangian is invariant are given by

$$\delta e_\mu^\alpha = i \bar{\epsilon}_{gAB} \gamma^\alpha \chi_\mu^{AB}, \quad (2.9)$$

$$\delta \chi_\mu^{AB} = \tilde{D}_\mu \epsilon_g^{AB}, \quad (2.10)$$

$$\begin{aligned} \delta B_\mu^A{}_B &= \frac{i}{e} (\bar{f}^{\nu AC} \gamma_\mu \gamma_\nu \epsilon_{gBC} - \bar{f}^{\nu BC} \gamma_\mu \gamma_\nu \epsilon_g^{AC}) \\ &\quad + \frac{i}{4} g_M^2 (\bar{\epsilon}_{BD} \gamma_\mu \Psi^{a(D} Z_a^{A)} - \bar{\epsilon}^{AD} \gamma_\mu \Psi_{a(D} \bar{Z}_{B)}^a) \\ &\quad - \frac{i}{2} g_M^2 (\bar{\epsilon}_g^{AC} \chi_{\mu DC} Z_a^D \bar{Z}_B^a - \bar{\epsilon}_{gBC} \chi_\mu^{DC} Z_a^A \bar{Z}_D^a) \\ &\quad + \frac{i}{8} g_M^2 \delta_B^A (\bar{\epsilon}_g^{EC} \chi_{\mu DC} - \bar{\epsilon}_{gDC} \chi_\mu^{EC}) Z_a^D \bar{Z}_E^a \\ &\quad + \frac{i}{8} g_M^2 (\bar{\epsilon}_g^{AD} \chi_{\mu BD} - \bar{\epsilon}_{gBD} \chi_\mu^{AD}) |Z|^2, \end{aligned} \quad (2.11)$$

$$\delta Z_a^A = i \bar{\epsilon}^{AB} \Psi_{Ba}, \quad (2.12)$$

$$\begin{aligned} \delta \Psi_{Bd} &= \gamma^\mu \epsilon_{AB} (\tilde{D}_\mu Z_d^A - i A \bar{\chi}_\mu^{AD} \Psi_{Dd}) \\ &\quad + \lambda f^{ab}{}_{cd} Z_c^C Z_b^D \bar{Z}_B^c \epsilon_{CD} - \lambda f^{ab}{}_{cd} Z_a^A Z_b^C \bar{Z}_C^c \epsilon_{AB} \\ &\quad + \frac{1}{4} g_M^2 Z_c^C Z_d^D \bar{Z}_B^c \epsilon_{CD} + \frac{1}{16} g_M^2 |Z|^2 Z_d^A \epsilon_{AB}, \end{aligned} \quad (2.13)$$

$$\begin{aligned} \delta \tilde{A}_\mu^c{}_d &= -i \lambda (\bar{\epsilon}_{AB} \gamma_\mu \Psi^{Aa} Z_b^B - \bar{\epsilon}^{AB} \gamma_\mu \Psi_{Ab} \bar{Z}_B^a) f^{bc}{}_{ad} \\ &\quad - 2i \lambda (\bar{\epsilon}_g^{AD} \chi_{\mu BD} - \bar{\epsilon}_{gBD} \chi_\mu^{AD}) Z_b^B \bar{Z}_A^a f^{bc}{}_{ad}, \end{aligned} \quad (2.14)$$

$$\begin{aligned} \delta C_\mu &= -iq g_M^2 (\bar{\epsilon}_{AB} \gamma_\mu \Psi^{Aa} Z_a^B - \bar{\epsilon}^{AB} \gamma_\mu \Psi_{Aa} \bar{Z}_B^a) \\ &\quad - 2iq g_M^2 (\bar{\epsilon}_g^{AD} \chi_{\mu BD} - \bar{\epsilon}_{gBD} \chi_\mu^{AD}) Z_a^B \bar{Z}_A^a, \end{aligned} \quad (2.15)$$

where $\epsilon_m^{AB} = A \epsilon_g^{AB} = \epsilon^{AB}$ (with $A^2 = \frac{1}{2}$).

In presenting the result of gauging the global symmetries of the M2-brane theories with six supersymmetries we have chosen to use the three-algebra formulation. The reason for this choice is not that it is more fundamental than the ABJM/ABJ quiver formulation (which probably is not the case) but rather one of convenience. The structure constants and its fundamental identity encode in a single form all possible gauge groups for $N = 6$ as classified by [12]. Indeed, the fact that the fundamental identity has exactly the same solutions as the quiver formulation has been shown by Palmkvist in [13]. Some implications of this fact will be discussed further in the next section.

⁵In this reference this result is presented as $q^2 = \frac{1}{16}$ but it is more appropriate to use "q" instead of its square as done in this presentation.

3. Comments

In this section we will elaborate on some of the properties of the topologically gauged M2-brane theories with six supersymmetries [16, 17] that were presented in some detail in the previous section.

3.1 The role of the structure constants in the three-algebra formulation

The structure constants used in [29, 28] are antisymmetric in both the upper and lower pair of indices and the corresponding fundamental identity reads

$$f^{e[a}{}_{dc} f^{b]d}{}_{gh} = f^{ab}{}_{d[g} f^{ed}{}_{h]c}. \quad (3.1)$$

There is a connection between this three-algebra and generalized Jordan triple systems as explained in [28, 13] but this will not be used here⁶. Instead we will discuss various forms for the structure constants that solve the fundamental identity and see what the implications are for the theory⁷. Recall first that a general way to obtain structure constants that satisfy the fundamental identity is to first replace each three-algebra index by a pair of indices by using the elements of the three-algebra T^a and T_a . Thus the scalar fields become (suppressing the R-symmetry index):

$$Z_a \rightarrow Z_a(T^a)_i^{i'} = Z_i^{i'}, \quad \bar{Z}^a \rightarrow \bar{Z}^a(T_a)_{i'}^i = \bar{Z}_{i'}^i, \quad (3.2)$$

and then set⁸

$$f^{ab}{}_{cd} \rightarrow f^{i'j'k'l'}_{i'j'k'l'} = \delta_k^i \delta_l^j \delta_{j'}^{k'} \delta_{i'}^{l'} - \delta_l^i \delta_k^j \delta_{i'}^{k'} \delta_{j'}^{l'}, \quad (3.3)$$

which is often used to rewrite the triple product as (using $X_a \rightarrow X_i^{i'}$ etc)

$$f^{ab}{}_{cd} X_a Y_b \bar{Z}^c \rightarrow [X, Y; \bar{Z}]_l^{l'} := (X \bar{Z} Y)_l^{l'} - (Y \bar{Z} X)_l^{l'}. \quad (3.4)$$

Note that the ranges of the two types of indices (primed and unprimed) are not related here and the same is true for the kinds of symmetry groups they transform under. This form of the structure constants thus corresponds to quivers with gauge groups like $SU(M) \times SU(N) \times U(1)$ which explains how both symmetric quivers like the ABJM theories and non-symmetric gauge group pairs as in the ABJ cases can be accommodated. One can also eliminate, e.g., the primed indices by letting them take only one value giving

$$f^{ab}{}_{cd} \rightarrow f^{ij}{}_{kl} = \delta_{kl}^{ij} = \delta_k^i \delta_l^j - \delta_l^i \delta_k^j, \quad (3.5)$$

which translates into

$$f^{ab}{}_{cd} X_a Y_b \bar{Z}^c \rightarrow (X \cdot \bar{Z}) Y_l - (Y \cdot \bar{Z}) X_l. \quad (3.6)$$

⁶It is interesting to note that the restriction of the generalized Jordan algebra (GJA) to the three-algebra structures found in M2-brane theories is not coming from the Chern-Simons term. In fact, the Chern-Simons term implies the GJA identity without restrictions.

⁷I am grateful to Jakob Palmkvist for discussions concerning the various forms of the structure constants mentioned here.

⁸The following form follows directly from the results in [29].

In this case the solution to the fundamental identity corresponds to only one non-abelian simple gauge group factor (times a $U(1)$ factor), or in other words a (complex) vector model in the language of sigma models. This is interesting since vector models have been studied a lot and, e.g., the fix-point structure is much better understood than for matrix-like quiver theories. This will be discussed in the last subsection below.

Of course, also the cases in the classification that involve symplectic groups can be accommodated. This is done by choosing the structure constants appropriately, namely

$$f^{ijkl} = J^{ik}J^{jl} - J^{jk}J^{il} - J^{ij}J^{kl}, \quad (3.7)$$

where J^{ij} is antisymmetric.

Finally, a free version of ABJM/ABJ corresponding to one M2-brane with six supersymmetries is obtained if the indices are chosen to have just one component which means that the structure constants vanish. Of course, the center of mass theory obtained in this case has additional supersymmetries adding up to eight.

The higgsing to be described below is independent of the structure constants and thus takes place in all the different cases discussed above in particular for the theory describing a single M2-brane. It would be interesting to find out what the proper string/M theory interpretation is of such a higgsing to AdS_3 which happens to be a chiral point super-TMG theory as we will see below.

3.2 The AdS_3 solution and the higgsing to a TMG theory at the chiral point with six supersymmetries

The bosonic part of the lagrangian reads

$$L = \frac{1}{g^2} L_{CS(\omega)} - eg^{\mu\nu} \partial_\mu Z_a^A \partial_\nu \bar{Z}_A^a - \frac{e}{8} |Z|^2 R - eV_{pot}(Z, \bar{Z}), \quad (3.8)$$

where $V_{pot}(Z, \bar{Z})$ is the six-order scalar potential and $L_{CS(\omega)}$ the gravitational Chern-Simons term expressed in terms of the spin-connection $\omega(e)$. By varying the action with respect to the complex scalar fields \bar{Z}_A^a we get the Klein-Gordon equation

$$\square Z_a^A - \frac{1}{8} R Z_a^A - \partial_{\bar{Z}_A^a} V_{pot}(Z, \bar{Z}) = 0, \quad (3.9)$$

while a variation with respect to the dreibein (or the metric) gives the Cotton equation

$$\begin{aligned} & \frac{1}{g^2} C_{\mu\nu} - \frac{e}{8} (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) |Z|^2 + \frac{e}{2} g_{\mu\nu} V_{pot} \\ & - e (\partial_\mu Z_a^A \partial_\nu \bar{Z}_A^a - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \partial_\rho Z_a^A \partial_\sigma \bar{Z}_A^a) + \frac{e}{8} (D_{(\mu} D_{\nu)} |Z|^2 - g_{\mu\nu} \square |Z|^2) = 0. \end{aligned} \quad (3.10)$$

If we trace the Cotton equation we can use the fact that the Cotton tensor has zero trace to get

$$\frac{1}{8} |Z|^2 R + 3V_{pot} = \frac{1}{2} (Z_a^A \square \bar{Z}_A^a + \bar{Z}_A^a \square Z_a^A). \quad (3.11)$$

Next we use the Klein-Gordon equation above to replace the RHS of the last equation by terms involving the curvature scalar and the potential. We find

$$3V_{pot} = \frac{1}{2} (Z \partial_Z + \bar{Z} \partial_{\bar{Z}}) V_{pot}. \quad (3.12)$$

Since the potential is homogeneous of degree three in both Z and \bar{Z} this equation becomes an identity showing that the KG and Cotton equations are compatible.

We can then easily verify that the above bosonic equations are solved by a scalar VEV v , i.e., $\langle Z_a^A \rangle = v\delta^{A4}\delta_{a8}$, and an AdS_3 background space-time satisfying

$$R_{AdS} = -24v^{-2}V_{pot}(v). \quad (3.13)$$

The extended potential appearing in the topologically gauged theory is crucial for the rest of the discussion so we present it in some detail here. Recall that there are terms of this kind already in the ungauged ABJM theory

$$V_{ABJM}^{(st)} = \frac{2e}{3}|\Upsilon^{CD}{}_{Bd}|^2, \quad \Upsilon^{CD}{}_{Bd} = \lambda f^{ab}{}_{cd}Z_a^C Z_b^D \bar{Z}_B^c + \lambda f^{ab}{}_{cd}\delta_B^{[C} Z_a^D] Z_b^E \bar{Z}_E^c.$$

In the three-algebra formulation these terms have two structure constants and correspond to a "single trace" [17] in the three-algebra. The new terms arising in the topological gauging [16] have either one structure constant corresponding to a "double trace" (dt)

$$V_{ABJM}^{(dt)} = \frac{e}{8}\lambda g_M^2 f^{ab}{}_{cd}|Z|^2 Z_a^C Z_b^D \bar{Z}_C^c \bar{Z}_D^d + \frac{e}{2}\lambda g_M^2 f^{ab}{}_{cd}Z_a^B Z_b^C (Z_e^D \bar{Z}_B^e) \bar{Z}_C^c \bar{Z}_D^d. \quad (3.14)$$

or no structure constant corresponding to a "triple trace" (tt)

$$V_{ABJM}^{(tt)} = \frac{5e}{12 \times 64}g_M^4(|Z|^2)^3 - \frac{e}{32}|Z|^2|Z|^4 + \frac{e}{48}|Z|^6, \quad (3.15)$$

where the double trace term $|Z|^4 = (Z_a^A \bar{Z}_B^a)(Z_b^B \bar{Z}_A^b)$ etc. One should note how the two parameters λ and g_M appear in these expressions [17].

The derivation of the lagrangian in this case has been checked for all terms in δL except a few multi-fermion non-derivative terms. The calculation involves a large number of cross checks on the coefficients appearing in both the lagrangian and extended transformation rules. Besides this fact the properties possessed by the last set of six-order potential terms for the complex ABJM scalar fields above strongly indicate that the construction is correct. In fact, collecting the relevant terms and inserting the VEV for the scalar fields [17]

$$Z^A = v\delta_4^A + z^A, \quad (3.16)$$

where for simplicity we have given this equation in the specific case of the $U(N) \times U(N)$ quiver version [17], that is, the VEV term is also diagonal in the two fundamental indices leading to an identification of the two gauge groups as in the original version of this Higgs effect [33].

We can now evaluate the potential for this VEV, where only the terms in the potential not containing structure constants contribute. We then find that

$$R_{AdS} = -24v^{-2}V_{pot}(v) = -24v^{-2}(\frac{1}{16^2}g_M^4 v^6) = -\frac{3}{32}(g_M v)^4, \quad (3.17)$$

which means that the three-dimensional cosmological constant is

$$\Lambda = -\frac{1}{64}(g_M v)^4. \quad (3.18)$$

Inserting the scalar VEV into the above bosonic lagrangian we find

$$L = \frac{1}{g_M^2} L_{CS}(\omega) - \frac{\epsilon}{8} v^2 R - e\left(\frac{1}{16^2} g_M^4 v^6\right). \quad (3.19)$$

By introducing the Newton's constant through $\frac{1}{8}v^2 = \kappa^{-2}$ and the cosmological length scale l by $\Lambda = -\frac{1}{l^2}$ we can compare our situation to that of Li, Song and Strominger [34] in their analysis of chiral gravity in three dimensions. However, since we have opposite signs for the Einstein-Hilbert and cosmological terms we should use the following lagrangian instead

$$L = \frac{1}{\kappa^2} \left(\frac{1}{\mu} L_{CS}(\omega) - (R - 2\Lambda) \right). \quad (3.20)$$

If we read off the values of these parameters in terms of our parameters v and g_M we find

$$\mu = \frac{g_M^2}{\kappa^2} = \frac{1}{8}(g_M v)^2, \quad l^{-2} = \frac{1}{64}(g_M v)^4, \quad (3.21)$$

i.e.,

$$\mu l = 1 \quad (3.22)$$

and the theory has thus been higgsed [16, 17] into a topologically massive AdS supergravity theory (TMG) [35] at the chiral point. This means that one needs to understand the implications of the negative energy black holes that appear as solution in this kind of theories and refer the reader to a recent discussion of some related issues by Deser and Franklin [36].

In the higgsed *AdS* phase there are six residual supersymmetries. These are obtained after the breaking of the superconformal symmetries as linear combinations of the original supersymmetries and superconformal symmetries. The combination chosen properly can be seen to generate the correct *AdS* covariant derivatives and to eliminate a term that otherwise would have been a Goldstone term in the transformation rules.

One should be able to follow all the degrees of freedom in the AdS_3 phase of the theory backwards to the pre-higgsed superconformal phase to conclude that there can not be any propagating gravitational modes in AdS_3 after the higgsing. This might, in fact, be considered to be the reason why the theory ends up at chiral point. At the chiral point there are problematic log-modes but off the chiral point there would have appeared massive gravitational modes that seem difficult to account for in our present situation. We have nothing to add concerning the log-modes but it might be interesting to study this issue in light of the connection of the chiral point to a conformal theory that one finds in the topologically gauged $N = 6$ M2-brane theories studied here. If this argument is correct one would expect the same chiral point phenomenon to occur in the topologically gauged BLG theory partly constructed in [14]. The answer to this question has, however, to await the construction of the complete lagrangian in that case (see [15]) and a proper analysis of the nature of the degrees of freedom.

In somewhat more detail the BLG situation is currently as follows. The coupling of the superconformal gravity theory to BLG was attempted in [14] using the Noether method. The calculations can preferably be organized in number of derivatives appearing in the terms in the supervariation of the lagrangian. The virtue of this approach is that the

variations of the Chern-Simons vector gauge fields need not be assumed but can instead be directly inferred from the calculation where the only input is the expressions for the variation of the matter fields (i.e spin 0 and 1/2). One then finds that it is necessary to introduce a number of new terms in the coupled lagrangian beyond the standard Noether term coupling the supercurrent and the Rarita-Schwinger field. For BLG this can be carried through in [14] cancelling all terms in δL at order three and two in covariant derivatives. At this order in derivatives the result was

$$\begin{aligned} L_{BLG}^{top} = & L_{grav}^{conf} + L_{BLG}^{cov} \\ & + \frac{1}{\sqrt{2}} ie \bar{\chi}_\mu \Gamma^i \gamma^\nu \gamma^\mu \Psi^a \tilde{D}_\nu X^{ia} \\ & - \frac{i}{4} \epsilon^{\mu\nu\rho} \bar{\chi}_\mu \Gamma^{ij} \chi_\nu (X_a^i \tilde{D}_\rho X_a^j) + \frac{i}{\sqrt{2}} \bar{f}^\mu \Gamma^i \gamma_\mu \Psi_a X_a^i \\ & - \frac{e}{16} X^2 \tilde{R} + \frac{i}{16} X^2 \bar{f}^\mu \chi_\mu, \end{aligned} \quad (3.23)$$

where i is an 8-dimensional vector R-symmetry index and f^{abcd} is the totally antisymmetric three-algebra structure constants. f^μ is the dualized Rarita-Schwinger field strength of the "spin 3/2" field χ_μ .

Interestingly enough the last two terms are exactly as expected from ordinary local scale invariance giving some hopes that this construction actually makes sense. It would be most welcome to find another approach to derive this theory. In [27] superspace methods⁹ were adopted but this has so far not led to any conclusive results concerning the existence of this theory. Since as we will see below, the ABJM construction works without any problems, one may hope that also the gauged BLG theory exists and that the full theory can be found¹⁰.

One can also in this case consider turning off the four-indexed structure constants and reduce the theory to that relevant for one brane. This theory should then, based on the degrees of freedom argument above, contain a potential with a similar structure as in the ABJM case. At least we expect the potential in the topologically gauged BLG theory to contain "triple trace" terms which give rise to a Higgs effect leading to a chiral point TMG theory and properties (physical modes etc) similar to those in the topologically gauged ABJM case.

3.3 Scaling limits and gravity free matter theories in curved backgrounds

Above we have discussed the result of the higgsing and checked explicitly that it leads to a chiral point supergravity of the TMG type. This statement was seen to be true for any value of the scalar VEV v , level $\lambda = \frac{2\pi}{k}$ and g_M introduced in the previous section. Thus it is of some interest to form physical constants by considering various combinations of these three parameters and to see how the physics depends on the scaling of the scalar VEV to infinity which in the ungauged case corresponds to going from M-theory to D2-branes in string theory. In fact, the higgsing done here turns out to be a rather straightforward

⁹This is closely related to previous work in [26]. See also [37, 38] for some more recent work relevant in this context.

¹⁰The complete theory has now been constructed, see [15].

generalization of the Higgs effect for Chern-Simons theories in three flat dimensions originally found by Mukhi and Papageorgakis [33]. In [17] we follow basically the same steps for our higgsed topologically gauged M2-theories with six supersymmetries and arrive at a lagrangian with the following structure, before taking the scaling limit,

$$\begin{aligned} L = & \frac{1}{\mu\kappa^2}L_{CD}(\omega) - (\frac{1}{\kappa^2} + \frac{z}{\kappa} + z^2)R + \frac{1}{g_{YM}^2}(F^+)^2 - DzD\bar{z} \\ & -(g_{YM}^2 z^4 + \text{subleading}) - (\mu g_{YM} z^3 + \text{subleading}) - (\frac{\mu^2}{\kappa^2} + \text{subleading}), \end{aligned} \quad (3.24)$$

where we have used the definitions

$$\kappa^2 \propto \frac{1}{v^2}, \quad g_{YM}^2 \propto (\lambda v)^2, \quad \mu \propto (g_M v)^2. \quad (3.25)$$

We now need a strategy to get a sensible result in the scaling limit $\lambda \rightarrow 0$ corresponding to the compactification circle shrinking to zero radius. Demanding that g_{YM} stays fixed means that $v \rightarrow \infty$. If we also want a fixed cosmological constant, which we saw above behaves as $\Lambda \propto \mu^2 \propto (g_M v)^4$, we need also $g_M \rightarrow 0$. This determines the scaling behaviour of all three parameters in the original lagrangian: $\lambda \propto v^{-1}$, or in fact the level goes to infinity as $k \propto v$, while κ and g_M go to zero as $\propto v^{-1}$. In the quiver version of this analysis carried out in [17] also N (from $U(N) \times U(N)$) enters and the 't Hooft parameter can be introduced.

This means that as the VEV diverges the AdS geometry stays fixed while the gravitational coupling constant κ goes to zero. Note that the unwanted linear term in the fluctuations of the scalar fields z that appears in the above lagrangian actually cancels a term that is subleading to the cosmological term on the last line. The end-result is thus a non-conformal matter theory consisting of scalars, spinors and Yang-Mills fields with six supersymmetries living in a fixed *AdS* geometry without gravity. Other methods to construct matter theories living in fixed non-trivial (i.e. not flat) geometries have been discussed recently by Festuccia and Seiberg [39], see also [40].

It might be interesting to note [17] that if the procedure just described had been carried out starting with the above lagrangian but now multiplied by g_M^2 the scaling limits inferred from taking the VEV v to infinity would be rather different. In this case both g_{YM} and Newton's constant κ can be kept fixed while the geometry becomes infinitely curved if we try to decouple gravity. The final result is thus in this case a supergravity theory that seems to make sense only for non-zero values of Newton's constant.

3.4 Indications of a "sequential AdS/CFT" phenomenon

The discussion in this section is based on the ABJM results presented in the previous section but relies heavily also on some more speculative points to be made precise below. We will try to argue that an explicit realization of what we will call "sequential AdS/CFT" might follow from the higgsing properties of topologically gauged ABJM theories explained above¹¹. This will require at least the following three assumptions: (1) topologically gauged M2 brane theories arise in *AdS*₄/*CFT*₃ from adopting Neumann boundary conditions instead of the usual Dirichlet ones, (2) it is possible to make sense of "going to the boundary

¹¹This should also apply to the topologically gauged BLG theory that has now been constructed [15].

twice”¹², perhaps by relating the higgsing (and the breaking of the symmetries of the CFT_3 to those of AdS_3) to a change of foliation of AdS_4 and (3) the TMG chiral supergravity in AdS_3 can be used in an AdS_3/CFT_2 context and thus has some CFT_2 on the boundary.

Some arguments in favor of these assumptions can be found in the literature although conclusive ones may not exist for any of the three assumptions. We will here discuss the assumptions in the context of topologically gauged ABJM theories in order to assess the possibilities to realize the “sequential AdS/CFT” in the specific case

$$AdS_4(N)/CFT_3(TG) \rightarrow AdS_3(H)/CFT_2 \quad (3.26)$$

where N refers to Neumann boundary conditions, TG to a topologically gauged ABJM/ABJ (or BLG) theory and H to its higgsed version. The final CFT_2 is unknown. Needless to say, if this sequence can be made sense of, a further extension to higher or lower dimensions would be very interesting (see e.g. [42, 43, 44]). Ideas somewhat related to those discussed here can be found in e.g. [19] and more recently in [25]. We will have reason to come back to the last work again below. There is also a speculation due to M. Vasiliev [22] based on algebraic higher spin arguments about the possibility to have more than one AdS/CFT following each other. However, all previous proposals lack a dynamical realization or mechanism for connecting two or more AdS/CFT correspondencies. The new ingredient here that perhaps can make this scenario more realistic is the conformal spontaneous symmetry breaking, ending in a theory in AdS_3 , triggered by the new terms in the scalar field potential generated by the topological gauging. Note that this discussion is relevant for stacks of branes as well as for a single one.

We now discuss the three assumptions in succession:

Assumption 1: Topologically gauged M2 brane theories arise in AdS_4/CFT_3 from adopting Neumann boundary conditions instead of the usual Dirichlet ones.

The issue of which boundary conditions to use and their relation to the terms that appear in the expansion of bulk fields in powers of the radial coordinate has been discussed both for scalars within the Breitenlohner-Freedman bound [45] and outside [18], as well as for Chern-Simons vector fields in three dimensions [46]. Unitarity is a central and unsolved issue in most discussions on Neumann boundary conditions, see for instance [47] and references therein. In the context of AdS_3/CFT_2 that will be addressed below, see also [48]. Attempts to generalize these results for spin 0 and 1 to spin 2 can be found in several papers where for instance the relation of the Cotton tensor to Neumann boundary conditions in AdS_4/CFT_3 is pointed out. Among the more explicit papers are [19, 21, 20]¹³. In fact, one may introduce [21] two CFT’s of opposite chirality as boundary field theories dual to the same AdS_4 bulk theory with Neumann boundary conditions which gives a possible explanation of the arbitrariness in the choice of chirality of the conformal gravity sector in the topological gauging procedure.

¹²Techniques like those discussed in [41] may be useful in this context.

¹³A possible connection to topologically gauged BLG theory was in fact mentioned already in the last of these references.

Concerning the relation between the Dirichlet and Neumann in the spin two case, one needs to understand how the interpretation of the zeroth and third order terms in the radial expansion of the Fefferman-Graham metric can be interchanged. Thus consider the metric (see e.g. [21])

$$ds^2 = \frac{l^2}{r^2}(dr^2 + (\eta_{ij} + h_{ij}(r, x))dx^i dx^j), \quad (3.27)$$

which, when inserted into the AdS_4 gravitational bulk equations, leads to an expansion in the radial coordinate

$$\bar{h}_{ij}(r, p) = (1 + \dots)h_{ij}^{(0)}(p) + ((pr)^3 + \dots)h_{ij}^{(3)}(p). \quad (3.28)$$

As explained in [21] using boundary conditions

$$\square^{1/2}h_{ij}^{(0)} = \pm\epsilon_{ikl}\partial_k h_{lj}^{(0)} \quad (3.29)$$

makes it possible to define a Legendre transformation related to the Cotton tensor. This Legendre transformation is supposed to take one from a (free) CFT_3 in the UV related to Dirichlet boundary conditions to two new (interacting) CFT_3 's of opposite chirality in the IR related to Neumann boundary conditions. What is needed for this to work is a third order derivative operator that can be inverted in the same sense as the Chern-Simons operator appearing in the usual superconformal M2 theories or in the work of Witten in [46]. That the linearized Cotton tensor has the wanted properties is demonstrated in [21]. This may also be seen by solving for all the components of the metric from the linearized inhomogeneous Cotton equation written out in the light-cone gauge [49]:

$$\begin{aligned} h_{++} &= -2\partial_-^{-3}T_{2-}, \\ h_{+2} &= -\partial_-^{-3}T_{--}, \\ \partial_+ h_{++} &= 2\partial_-^{-2}T_{2+} - 2\partial_-^{-3}(\partial_2 T_{22}), \\ \partial_+ h_{+2} &= \frac{1}{2}\partial_-^{-2}T_{22} - \partial_-^{-3}(\partial_2 T_{2-}), \\ \partial_+^2 h_{+2} &= -\partial_-^{-1}T_{++} + \partial_-^{-3}(\partial_2^2 T_{22}), \end{aligned} \quad (3.30)$$

where $T_{\mu\nu}$ with $\mu = +, -, 2$ is the light-cone components of the stress tensor for the matter system in question. This, in fact, also shows that the topologically gauged theories do not contain any new propagating degrees of freedom in three dimensions as a result of the gauging. The spin two Legendre transformation

$$\tilde{W}(\tilde{h}) = W(h) + V(\tilde{h}, h) \quad (3.31)$$

is generated by the second variation of the gravitational Chern-Simons term

$$V(\tilde{h}, h) = -\frac{l^2}{2\kappa^2} \int d^3x h_{\mu\nu} \frac{\delta^2 S_{(CS)}}{\delta g_{\mu\nu} \delta g_{\rho\sigma}} \tilde{h}_{\rho\sigma}, \quad (3.32)$$

where h is the metric fluctuation in the Dirichlet case and \tilde{h} in the Neumann case. W and \tilde{W} are the on-shell actions in the two cases.

The situation described above should be compared to the simpler case of the $O(N)$ model where things are better controlled [50, 51, 52, 53, 54, 55]. The UV fixed point is then described by a free theory while the non-trivial fix point is in the IR and interacting but weakly so. Furthermore, there is an explicitly known Legendre transformation [56, 57] between these two fix points theories whose bulk dual is also understood to some extent in terms of Vasiliev's higher spin (HS) theory [58]. It is interesting to note that the HS bulk theory dual to the free UV CFT_3 has massless gauge fields of all spins from two and up while the interacting IR CFT_3 is a higgsed HS theory where all fields with spin above two have become massive [56, 60, 61]. Recent work supporting this picture can be found, e.g., in [62].

More recently other indications of HS duals of interacting fix point theories have appeared in [63]. Here one is analyzing scalar $O(N)$ and $U(N)$ models in the singlet sector, implemented by coupling the theory to a gauge theory of Chern-Simons type in order to add only trivial operators¹⁴. In perturbation theory in the 't Hooft coupling $\lambda = 4\pi \frac{N}{k}$ one finds that all the HS currents are preserved at infinite N even at the interacting fix point although they are broken by $\frac{1}{N}$ effects. Related results are found for fermions in [64]. The interesting implication of these results seems to be that there should exist new HS theories beyond the known Vasiliev type theories in AdS_4 [58, 59] related to the known ones by some kind of deformation. Another intriguing result in this context is the appearance of lines of fixed points similar to what generically happens in two-dimensional conformal field theory. For a recent account of some higher spin issues in this context, see [65].

In the case of the $O(N)$ models with their weak-weak duality one may actually hope to be able to prove the AdS/CFT correspondence. One such attempt is advocated in [54, 55] based on bilocal "collective fields" following on ideas that started with Sundborg [50], Witten [51], and Sezgin and Sundell [52].

Assumption 2: The symmetry breaking occurring in the topologically gauged ABJM/ABJ theories bringing the theory from a CFT_3 to a (chiral) TMG theory in AdS_3 corresponds to a change of foliation in AdS_4

The higgsing that takes place in the topologically gauged M2-brane theories described in the previous section breaks the three-dimensional superconformal symmetry to that of AdS_3 . This phenomenon¹⁵ was noticed in [16], and analyzed in detail for some specific cases in [17]. It follows from combining the effect of the conformal coupling term of two scalar fields and the scalar curvature with the last set of potential terms that are independent of the structure constants. What is found is that the theory has an AdS_3 solution that sits exactly at a chiral point similar to that discussed by Li, Song and Strominger [34] but with signs corresponding to TMG. This changes the sign of the energies of the black holes and the would-be propagating physical gravity modes away from the chiral point. This thus leads to the appearance of negative energy black holes which constitute a potential

¹⁴This is the same argument as used in this paper for gravitational theories.

¹⁵This symmetry breaking can also be viewed as the result of a gauge choice as shown in [66].

problem for the theories discussed here. What this means for the log-gravity/cft issue if anything is not clear.

The higgsing also involves finding the correct super- AdS_3 symmetry generators after breaking as combinations of the ones in the superconformal algebra. By checking the supersymmetry transformation rules after breaking one can infer the answer. One finds for instance that the covariant derivative in the conformal theory gets augmented by a gamma matrix term in a way that is familiar from any supergravity theory in AdS . This way of relating conformal and AdS symmetry algebras has been discussed, e.g., by M. Vasiliev in [22] based on HS algebra arguments, however, without having a theory exhibiting an appropriate Higgs phenomenon like the one we find in the topologically gauged ABJM theories constructed in [16].

In the context of AdS_4/CFT_3 the symmetry breaking taking place in the boundary theory requires an interpretation in the AdS_4 bulk theory. One possible such interpretation could be that it corresponds to a shift of foliation in the bulk. In fact it is well known that one can foliate AdS spaces in a number of different ways some leading directly to a boundary also with AdS geometry [23]. For an explicit application to AdS_4 , see [24] where for instance the following case is discussed:

$$ds^2 = \frac{1}{k + \frac{r^2}{L^2}} dr^2 + (k + \frac{r^2}{L^2}) L^2 d\Omega^2 + r^2 d\tilde{\Omega}^2, \quad (3.33)$$

where $k = \pm 1$ and $d\Omega^2$ and $d\tilde{\Omega}^2$ are metrics that can be either on a spherical or a hyperbolic space of arbitrary dimensions. This metric may be compared to the usual AdS/CFT metric which if written with the same radial coordinate reads

$$ds^2 = \frac{L^2}{r^2} dr^2 + r^2 dx_i^2. \quad (3.34)$$

Here the index on the coordinates x_i extends over the number of flat dimensions corresponding to both the spherical and hyperbolic spaces above. There are a number of questions concerning the nature of the boundaries in these cases and how they relate to each other. In fact, non-standard foliations could be problematic for unitarity, see [67].

Assumption 3: The TMG chiral supergravity makes sense in the AdS_3/CFT_2 context and thus has a CFT_2 on the boundary

The kind of CFT_2 that would be relevant for topologically gauged M2-branes is probably not yet discussed in the literature. We will therefore just mention some results that are well-known and that hopefully will have generalizations useful in our context. The question of the role of the chiral point in AdS_3/CFT_2 has been analyzed in the non-supersymmetric case by Skenderis et al in [68] where also the issue of logarithmic modes was addressed. In a more general context there has recently been some important progress [69, 70, 71, 72] concerning an AdS/CFT connection between \mathcal{W}_N CFT's in two dimensions and HS theories in AdS_3 of the Vasiliev type, see e.g. [73, 74]. The massless HS fields play here a crucial role since they correspond to the set of conserved HS currents in the \mathcal{W}_N CFT's, where N

can be any finite number ≥ 2 in this case. These Vasiliev type HS theories contain also two free massive scalar fields which make the theory non-conformal. It might be interesting to investigate if there is also here a conformal phase in the bulk and what field theory it might correspond to on the boundary.

Having discussed if the higgsed topologically gauged ABJM theory in AdS_3 may have a conformal boundary field theory in two dimensions, one could also ask if this theory in three dimensions can be associated with a boundary field theory even before higgsing, that is, as a conformal theory in three dimensions. Questions of this kind have indeed been addressed recently in the simpler setting of pure conformal gravity, see [75, 76].

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